

The appearance of prime manifolds in $\mathbf{C}^{(30)}$ with respect to complexity and geometry (see [B.Martelli - C.Petronio, *Complexity of geometric three-manifolds*, Geom. Dedicata **108** (2004), 15-69] for cases up to complexity 9, and [B.Martelli, *Complexity of 3-manifolds*, to appear in “Spaces of Kleinian groups”, London Math. Soc. Lecture Notes Ser. 329 (2006), Math.GT/0405250] for the last case) is summarized in the following table, where the symbol x/n means that x 3-manifolds appear in $\mathbf{C}^{(30)}$, among the n ones having the appropriate complexity and geometry, and bold character is used to indicate that all manifolds of the considered type appear in the catalogue:

complexity	1	2	3	4	5	6	7	8	9	10
lens	2/2	3/3	6/6	10/10	0/20	0/36	0/72	0/136	0/272	0/528
other elliptic	-	1/1	1/1	4/4	11/11	14/25	0/45	0/78	0/142	0/270
euclidean	-	-	-	-	-	6/6	-	-	-	-
Nil	-	-	-	-	-	7/7	3/10	2/14	0/15	0/15
$H^2 \times S^1$	-	-	-	-	-	-	-	0/2	-	0/8
SL_2R	-	-	-	-	-	-	13/39	5/162	2/513	0/1416
Sol	-	-	-	-	-	-	4/5	2/9	0/23	0/39
non-geometric	-	-	-	-	-	-	4/4	1/35	2/185	0/777
hyperbolic	-	-	-	-	-	-	-	-	2/4	1/25
TOTAL	2/2	4/4	7/7	14/14	11/31	27/74	24/175	10/436	6/1154	1/3078

Table 3: prime 3-manifolds (with complexity $c \neq 0$) involved in $\mathbf{C}^{(30)}$

We think worth noting that, for any fixed complexity c , catalogues $\mathcal{C}^{(2p)}$ cover, for increasing p , first the most “complicated” types of complexity c 3-manifolds and then the simplest ones. In fact, a comparative analysis of both complexity and geometric properties of manifolds represented by subsequent catalogues $\mathcal{C}^{(2p)}$, $1 \leq p \leq 15$ yields the following data:

complexity 1:

- all 2 lens spaces appear in $\mathcal{C}^{(16)}$.

complexity 2:

- all 3 lens spaces appear in $\mathcal{C}^{(20)}$;
- the only elliptic 3-manifold appears in $\mathcal{C}^{(18)}$.

complexity 3:

- all 6 lens spaces appear in $\mathcal{C}^{(24)}$;
- the only elliptic 3-manifold appears in $\mathcal{C}^{(20)}$.

complexity 4:

- all 10 lens spaces appear in $\mathcal{C}^{(28)}$;

- all 4 elliptic 3-manifolds appear in $\mathcal{C}^{(22)} \cup \mathcal{C}^{(24)}$.

complexity 5:

- none of the 20 lens spaces appear in $\mathbf{C}^{(30)}$;
- all 11 elliptic 3-manifolds appear in $\mathcal{C}^{(24)} \cup \mathcal{C}^{(26)} \cup \mathcal{C}^{(28)}$.

complexity 6:

- none of the 36 lens spaces appear in $\mathbf{C}^{(30)}$;
- 14 among 25 elliptic 3-manifolds appear in $\mathbf{C}^{(30)}$ (more precisely: 1 appears in $\mathcal{C}^{(24)}$, 3 appear in $\mathcal{C}^{(26)}$ and 10 appear in $\mathcal{C}^{(28)}$);
- all 6 euclidean 3-manifolds appear in $\mathcal{C}^{(24)} \cup \mathcal{C}^{(26)}$;
- all 7 Nil 3-manifolds appear in $\mathcal{C}^{(28)} \cup \mathcal{C}^{(30)}$.

complexity 7:

- none of the 72 lens spaces appear in $\mathbf{C}^{(30)}$;
- none of the 45 elliptic 3-manifolds appear in $\mathbf{C}^{(30)}$;
- 3 among 10 Nil 3-manifolds appear in $\mathbf{C}^{(30)}$ (more precisely: 2 appear in $\mathcal{C}^{(28)}$ and 2 appear in $\mathcal{C}^{(30)}$);
- 13 among 39 SL_2R 3-manifolds appear in $\mathbf{C}^{(30)}$ (more precisely: 7 appear in $\mathcal{C}^{(28)}$ and 6 appear in $\mathcal{C}^{(30)}$);
- 4 among 5 Sol 3-manifolds appear in $\mathbf{C}^{(30)}$ (more precisely: 2 - which are of type $TB(A)$ - appear in $\mathcal{C}^{(28)}$ and 2 - which are of type $KB(A)$ - appear in $\mathcal{C}^{(30)}$);
- all 4 non-geometric graph-manifolds appear in $\mathcal{C}^{(30)}$.

complexity 8:

- none of the 136 lens spaces appear in $\mathbf{C}^{(30)}$;
- none of the 78 elliptic 3-manifolds appear in $\mathbf{C}^{(30)}$;
- 2 among 14 Nil 3-manifolds appear in $\mathbf{C}^{(30)}$;
- none of the 2 $H^2 \times S^1$ 3-manifolds appear in $\mathbf{C}^{(30)}$;
- 5 among 162 SL_2R 3-manifolds appear in $\mathbf{C}^{(30)}$;
- 2 among 9 Sol 3-manifolds appear in $\mathbf{C}^{(30)}$ (more precisely: they both are of type $TB(A)$ and appear in $\mathcal{C}^{(30)}$);

- 1 among 35 non-geometric graph-manifolds appears in $\mathcal{C}^{(30)}$.

complexity 9:

- none of the 272 lens spaces appear in $\mathbf{C}^{(30)}$;
- none of the 142 elliptic 3-manifolds appear in $\mathbf{C}^{(30)}$;
- none of the 15 Nil 3-manifolds appear in $\mathbf{C}^{(30)}$;
- 2 among 513 SL_2R 3-manifolds appear in $\mathbf{C}^{(30)}$;
- none of the 23 Sol 3-manifolds appear in $\mathbf{C}^{(30)}$;
- 2 among 185 non-geometric graph-manifolds appears in $\mathcal{C}^{(30)}$;
- 2 among 4 hyperbolic 3-manifolds appear in $\mathcal{C}^{(30)}$.

complexity 10:

- none of the 528 lens spaces appear in $\mathbf{C}^{(30)}$;
- none of the 270 elliptic 3-manifolds appear in $\mathbf{C}^{(30)}$;
- none of the 15 Nil 3-manifolds appear in $\mathbf{C}^{(30)}$;
- none of the 8 $H^2 \times S^1$ 3-manifolds appear in $\mathbf{C}^{(30)}$;
- none of the 1416 SL_2R 3-manifolds appear in $\mathbf{C}^{(30)}$;
- none of the 39 Sol 3-manifolds appear in $\mathbf{C}^{(30)}$;
- none of the 777 non-geometric graph-manifolds appear in $\mathcal{C}^{(30)}$;
- 1 among 25 hyperbolic 3-manifolds appear in $\mathcal{C}^{(30)}$.