

TABLE 2

minimal gem	prime orientable 3-manifold M^3	$H_1(M^3)$	geometry	position in [M]
r_{1203}^{30}	S^3/D_{80}	Z_{16}	S^3	6 ₄₄
r_{1053}^{30}	S^3/D_{112}	Z_{16}	S^3	6 ₄₈
r_{18250}^{30}	$S^3/(Q_{28} \times Z_5)$	Z_{20}	S^3	6 ₄₉
r_{21444}^{30}	$S^3/(Q_{32} \times Z_5)$	$Z_2 + Z_{10}$	S^3	6 ₅₁
r_{1045}^{30}	$S^3/(P_{48} \times Z_{11})$	Z_{22}	S^3	6 ₅₇
r_{1035}^{30}	$S^3/(P_{48} \times Z_5)$	Z_{10}	S^3	6 ₅₆
r_{1040}^{30}	$S^3/(P_{48} \times Z_7)$	Z_{14}	S^3	6 ₅₅
r_{19178}^{30}	$S^3/(P_{120} \times Z_{23})$	Z_{23}	S^3	6 ₆₁
r_{17733}^{30}	$S^3/(P_{120} \times Z_{17})$	Z_{17}	S^3	6 ₆₀
r_{1122}^{30}	$S^3/(P_{120} \times Z_{13})$	Z_{13}	S^3	6 ₅₉
r_{21303}^{30}	$(S^2, (2, 1), (3, 1), (7, 3), (1, -1))$	Z_{11}	$SL_2(\mathbb{R})$	7 ₁₂₀
r_{17842}^{30}	$(S^2, (2, 1), (3, 1), (8, 1), (1, -1))$	Z_2	$SL_2(\mathbb{R})$	8 ₂₂₆
r_{21350}^{30}	$(S^2, (2, 1), (3, 1), (8, 3), (1, -1))$	Z_{10}	$SL_2(\mathbb{R})$	7 ₁₂₇
r_{28623}^{30}	$(S^2, (2, 1), (3, 1), (9, 2), (1, -1))$	Z_3	$SL_2(\mathbb{R})$	8 ₂₃₁
r_{44846}^{30}	$(S^2, (2, 1), (3, 1), (11, 2), (1, -1))$	0	$SL_2(\mathbb{R})$	8 ₂₄₃
r_{17755}^{30}	$(S^2, (2, 1), (4, 1), (5, 2), (1, -1))$	Z_6	$SL_2(\mathbb{R})$	7 ₁₃₄

(Table 2 continues...)

minimal gem	prime orientable 3-manifold M^3	$H_1(M^3)$	geometry	position in [M]
r_{45301}^{30}	$(\mathbb{S}^2, (2, 1), (4, 1), (7, 2), (1, -1))$	\mathbb{Z}_2	$SL_2(\mathbb{R})$	8 ₂₇₁
r_{48748}^{30}	$(\mathbb{S}^2, (2, 1), (5, 1), (5, 1), (1, -1))$	\mathbb{Z}_5	$SL_2(\mathbb{R})$	8 ₂₈₃
r_{48763}^{30}	$(\mathbb{S}^2, (2, 1), (5, 2), (5, 2), (1, -1))$	\mathbb{Z}_{15}	$SL_2(\mathbb{R})$	7 ₁₃₈
r_{19485}^{30}	$(\mathbb{S}^2, (3, 2), (3, 2), (3, 2), (1, -1))$	$\mathbb{Z}_3 + \mathbb{Z}_9$	<i>Nil</i>	6 ₆₇
r_{15814}^{30}	$(\mathbb{S}^2, (3, 1), (3, 2), (3, 2), (1, -1))$	$\mathbb{Z}_3 + \mathbb{Z}_6$	<i>Nil</i>	6 ₆₆
r_{20091}^{30}	$TB \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$	$\mathbb{Z}_3 + \mathbb{Z} + \mathbb{Z}$	<i>Nil</i>	8 ₃₇₇
r_{56760}^{30}	$(T, (2, 1))$	$\mathbb{Z} + \mathbb{Z}$	$SL_2(\mathbb{R})$	9 ₉₀₂
r_{21193}^{30}	$(\mathbb{RP}^2, (2, 1), (3, 1))$	\mathbb{Z}_{24}	$SL_2(\mathbb{R})$	7 ₁₆₄
r_{21188}^{30}	$(\mathbb{RP}^2, (2, 1), (3, 2))$	\mathbb{Z}_{24}	$SL_2(\mathbb{R})$	7 ₁₆₅
r_{20090}^{30}	$TB \begin{pmatrix} -1 & 0 \\ 3 & -1 \end{pmatrix}$	$\mathbb{Z}_4 + \mathbb{Z}$	<i>Nil</i>	8 ₃₉₂
r_{56762}^{30}	$(K, (2, 1))$	$\mathbb{Z}_8 + \mathbb{Z}$	$SL_2(\mathbb{R})$	9 ₉₄₁
r_{17038}^{30}	$TB \begin{pmatrix} -4 & 1 \\ -1 & 0 \end{pmatrix}$	$\mathbb{Z}_6 + \mathbb{Z}$	<i>Sol</i>	8 ₃₉₃
r_{17043}^{30}	$TB \begin{pmatrix} 4 & -1 \\ 1 & 0 \end{pmatrix}$	$\mathbb{Z}_2 + \mathbb{Z}$	<i>Sol</i>	8 ₃₉₄
r_{56755}^{30}	$(\mathbb{A}, (2, 1), (1, -2)) \cup \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} (\mathbb{A}, (2, 1), (1, -2))$	$\mathbb{Z}_7 + \mathbb{Z}$	-	9 ₉₅₂
r_{56759}^{30}	$(\mathbb{A}, (2, 1), (1, -1)) \cup \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} (\mathbb{A}, (2, 1), (1, -1))$	$\mathbb{Z}_5 + \mathbb{Z}$	-	9 ₉₅₀

(Table 2 continues...)

minimal gem	prime orientable 3-manifold M^3	$H_1(M^3)$	geometry	position in [M]
r_{21476}^{30}	$(K \tilde{\times} I) \cup (K \tilde{\times} I) / \begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix}$	$\mathbb{Z}_2 + \mathbb{Z}_2 + \mathbb{Z}_4$	<i>Sol</i>	7 ₁₇₁
r_{45716}^{30}	$(K \tilde{\times} I) \cup (K \tilde{\times} I) / \begin{pmatrix} -1 & -1 \\ 1 & 2 \end{pmatrix}$	$\mathbb{Z}_4 + \mathbb{Z}_4$	<i>Sol</i>	7 ₁₆₉
r_{19144}^{30}	$(\mathbb{D}, (2, 1), (2, 1), (1, 0)) \cup \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} (\mathbb{D}, (2, 1), (3, 2), (1, 0))$	\mathbb{Z}_4	-	7 ₁₇₃
r_{18104}^{30}	$(\mathbb{D}, (2, 1), (2, 1), (1, 0)) \cup \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} (\mathbb{D}, (2, 1), (3, 2), (1, -1))$	\mathbb{Z}_{20}	-	7 ₁₇₅
r_{19251}^{30}	$(\mathbb{D}, (2, 1), (2, 1), (1, 0)) \cup \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} (\mathbb{D}, (2, 1), (3, 1), (1, -1))$	$\mathbb{Z}_2 + \mathbb{Z}_2$	-	8 ₄₁₀
r_{19087}^{30}	$(\mathbb{D}, (2, 1), (2, 1), (1, 0)) \cup \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} (\mathbb{D}, (2, 1), (3, 1), (1, -1))$	\mathbb{Z}_{28}	-	7 ₁₇₄
r_{1111}^{30}	$(\mathbb{D}, (2, 1), (2, 1), (1, 0)) \cup \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} (\mathbb{D}, (2, 1), (3, 1), (1, 0))$	\mathbb{Z}_4	-	7 ₁₇₂
r_{56897}^{30}	$Q_1(2, -3)$	$\mathbb{Z}_5 + \mathbb{Z}_5$	H^3	9 ₁₁₅₁
r_{45332}^{30}	$Q_4(2, -1)$	$\mathbb{Z}_3 + \mathbb{Z}_6$	H^3	9 ₁₁₅₃
r_{56912}^{30}	$Q_{10}(2, -1)$	$\mathbb{Z}_3 + \mathbb{Z}_9$	H^3	10 ₃₀₆₃

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Remark:

Note that the two Sol manifolds of type $(K \tilde{\times} I) \cup (K \tilde{\times} I) / A$ with minimal gems r_{21476}^{30} and r_{45716}^{30} have also the structure of (geometric) graph manifolds: they are $(\mathbb{D}, (2, 1), (2, 1), (1, -1)) \cup \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} (\mathbb{D}, (2, 1), (2, 1), (1, -1))$ and $(\mathbb{D}, (2, 1), (2, 1), (1, 0)) \cup \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} (\mathbb{D}, (2, 1), (2, 1), (1, 1))$.