TABLE 1

minimal gem	**prime orientable 3-manifold $\mathbf{M^3}$**	$\mathbf{H_1(M^3)}$	**geometry**	**upper bound for** $\mathbf{c'_{GM}(M^3)}$	$\mathbf{c(M^3)}$
r_1^2	$\mathbb{S}^3$	0	S^3	0	0
r_1^8	$L(2,1)$	$\mathbb{Z}_2$	”	0	0
r_1^{12}	$L(3,1)$	$\mathbb{Z}_3$	”	0	0
r_1^{16}	$L(5,2)$	$\mathbb{Z}_5$	”	1	1
r_2^{16}	$L(4,1)$	$\mathbb{Z}_4$	”	1	1
r_1^{18}	$QUAT = \mathbb{S}^3/Q_8$	$\oplus_2\mathbb{Z}_2$	”	2	2
r_1^{20}	$L(8,3)$	$\mathbb{Z}_8$	”	2	2
r_2^{20}	$L(5,1)$	$\mathbb{Z}_5$	”	2	2
r_4^{20}	$S^3/ < 3,2,2 >= \mathbb{S}^3/Q_{12}$	$\mathbb{Z}_4$	”	3	3
r_5^{20}	$L(7,2)$	$\mathbb{Z}_7$	”	2	2
r_9^{20}	$\mathbb{S}^1 \times \mathbb{S}^2$	$\mathbb{Z}$	$S^2 \times \mathbb{R}$	0	0
r_1^{22}	$BINTET = \mathbb{S}^3/P_{24}$	$\mathbb{Z}_3$	S^3	4	4
r_2^{22}	$S^3/(C_3 \times_i C_8) = \mathbb{S}^3/D_{24}$	$\mathbb{Z}_8$	”	4	4

(Table 1 continues...)

gem	3-manifold $\mathbf{M^3}$	$\mathbf{H_1(M^3)}$	geometry	$\mathbf{c_{GM}(M^3)}$	$\mathbf{c(M^3)}$
r_1^{24}	$EUCLID_0 = TB\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$	$\oplus_3\mathbb{Z}$	E^3	6	6
r_2^{24}	$BINDOD = \mathbb{S}^3/P_{120}$	0	S^3	5	5
r_4^{24}	$S^3/(C_5 \times_i C_8) = \mathbb{S}^3/D_{40}$	$\mathbb{Z}_8$	"	5	5
r_5^{24}	$EUCLID_1 = (K\tilde{\times}I) \cup (K\tilde{\times}I)/\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$	$\oplus_2\mathbb{Z}_4$	E^3	6	6
r_6^{24}	$EUCLID_3 = TB\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$	$\oplus_2\mathbb{Z}_2 \oplus \mathbb{Z}$	E^3	6	6
r_7^{24}	$EUCLID_2 = TB\begin{pmatrix}0 & 1\\ -1 & -1\end{pmatrix}$	$\mathbb{Z} \oplus \mathbb{Z}_3$	E^3	6	6
r_{13}^{24}	$S^3/(C_3 \times Q_8) = \mathbb{S}^3/(Q_8 \times \mathbb{Z}_3)$	$\mathbb{Z}_2 \oplus \mathbb{Z}_6$	S^3	4	4
r_{14}^{24}	$L(9,2)$	$\mathbb{Z}_9$	"	3	3
r_{21}^{24}	$L(10,3)$	$\mathbb{Z}_{10}$	"	3	3
r_{22}^{24}	$L(11,3)$	$\mathbb{Z}_{11}$	"	3	3
r_3^{24}	$L(13,5)$	$\mathbb{Z}_{13}$	"	3	3
r_{28}^{24}	$BINOCT = \mathbb{S}^3/P_{48}$	$\mathbb{Z}_2$	"	5	5
r_{32}^{24}	$L(12,5)$	$\mathbb{Z}_{12}$	"	3	3
r_{33}^{24}	$L(6,1)$	$\mathbb{Z}_6$	"	3	3
r_{154}^{24}	$\mathbb{S}^3/Q_{16}$	$\oplus_2\mathbb{Z}_2$	"	4	4
r_4^{26}	$S^3/(Q_8 \times_3 C_{15}) = \mathbb{S}^3/(P_{24} \times \mathbb{Z}_5)$	$\mathbb{Z}_{15}$	"	6	5
r_5^{26}	$E_2(0,2) = (K\tilde{\times}I) \cup (K\tilde{\times}I)/\begin{pmatrix}1 & 1\\ 1 & 0\end{pmatrix}$	$\oplus_2\mathbb{Z}_4$	Nil	6	6
r_6^{26}	$S^3/(C_5 \times_i C_{12}) = \mathbb{S}^3/(Q_{20} \times \mathbb{Z}_3)$	$\mathbb{Z}_{12}$	S^3	5	5
r_8^{26}	$S^3/(Q_8 \times_3 C_9) = \mathbb{S}^3/P'_{72}$	$\mathbb{Z}_9$	"	5	5
r_{10}^{26}	$S^3/(C_7 \times < 5,3,2 >) = \mathbb{S}^3/(P_{120} \times \mathbb{Z}_7)$	$\mathbb{Z}_7$	"	6	6
r_{11}^{26}	$EUCLID_5 = TB\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$	$\mathbb{Z} \oplus \mathbb{Z}_2$	E^3	6	6
r_{13}^{26}	$E_2(2,1) = TB\begin{pmatrix}-1 & 0\\ -1 & -1\end{pmatrix}$	$\mathbb{Z} \oplus \mathbb{Z}_4$	Nil	6	6
r_{14}^{26}	$TB\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}$	$\oplus_2\mathbb{Z}$	Nil	6	6
r_{31}^{26}	$EUCLID_4 = TB\begin{pmatrix}1 & -1\\ 1 & 0\end{pmatrix}$	$\mathbb{Z}$	E^3	6	6
r_{65}^{26}	$S^3/(C_3 \times Q_{16}) = \mathbb{S}^3/(Q_{16} \times \mathbb{Z}_3)$	$\mathbb{Z}_2 \oplus \mathbb{Z}_6$	S^3	5	5

(Table 1 continues...)

gem	**3-manifold $\mathbf{M^3}$**	$\mathbf{H_1(M^3)}$	**geometry**	$\mathbf{c_{GM}(M^3)}$	$\mathbf{c(M^3)}$
r_1^{28}	$[24, S_2^1] = (\mathbb{RP}^2,\ (2,1),(3,-1))$	$\mathbb{Z}_{24}$	$SL_2\mathbb{R}$	8	7
r_2^{28}	$[3, 5^2, S_6^5] = (\mathbb{S}^2, (3,1),(3,1),(5,-3))$	$\mathbb{Z}_3$	"	8	7
r_3^{28}	$[2^2, 3^2 \times 6, S_{10}^9] =$ $= (\mathbb{S}^2,\ (2,1),\ (2,1),\ (2,1),\ (3,-4))$	$\oplus_2\mathbb{Z}_2$	$SL_2\mathbb{R}$	8	7
r_5^{28}	$TB\begin{pmatrix} 0 & 1 \\ -1 & 3 \end{pmatrix}$	$\mathbb{Z}$	Sol	7	7
r_6^{28}	$[2^2 \times 4, S_1^4] = (K\tilde{\times}I) \cup (K\tilde{\times}I)/\begin{pmatrix} -1 & 0 \\ -1 & 1 \end{pmatrix}$	$\oplus_2\mathbb{Z}_2 \oplus \mathbb{Z}_4$	Nil	6	6
r_7^{28}	$S^3/(C_5 \times Q_8) = \mathbb{S}^3/(Q_8 \times \mathbb{Z}_5)$	$\mathbb{Z}_2 \oplus \mathbb{Z}_{10}$	S^3	5	5
r_9^{28}	$[3, 4^2, S_3^4] = (\mathbb{S}^2, (3,1),(3,1),(4,-3))$	$\mathbb{Z}_3$	$SL_2\mathbb{R}$	7	7
r_{10}^{28}	$TB\begin{pmatrix} 0 & 1 \\ -1 & -3 \end{pmatrix}$	$\mathbb{Z} \oplus \mathbb{Z}_5$	Sol	7	7
r_{13}^{28}	$L(21,8)$	$\mathbb{Z}_{21}$	S^3	4	4
r_{14}^{28}	$L(16,7)$	$\mathbb{Z}_{16}$	"	4	4
r_{19}^{28}	$[4^2, S_1^8] = (\mathbb{RP}^2, (2,1),(2,3))$	$\oplus_2\mathbb{Z}_4$	Nil	7	7
r_{27}^{28}	$S^3/(C_3 \times_i C_{16}) = \mathbb{S}^3/D_{48}$	$\mathbb{Z}_{16}$	S^3	5	5
r_{29}^{28}	$S^3/(C_3 \times_i C_{20}) = \mathbb{S}^3/(Q_{12} \times \mathbb{Z}_5)$	$\mathbb{Z}_{20}$	"	5	5
r_{33}^{28}	$L(13,3)$	$\mathbb{Z}_{13}$	"	4	4
r_{34}^{28}	$[3^2, S_1^3] = (\mathbb{S}^2, (3,2),(3,1),(3,-2))$	$\oplus_2\mathbb{Z}_3$	Nil	6	6
r_{41}^{28}	$L(19,7)$	$\mathbb{Z}_{19}$	S^3	4	4
r_{42}^{28}	$TB\begin{pmatrix} -1 & 0 \\ -2 & -1 \end{pmatrix}$	$\oplus_2\mathbb{Z}_2 \oplus \mathbb{Z}$	Nil	7	7
r_{49}^{28}	$L(11,5) = L(11,2)$	$\mathbb{Z}_{11}$	S^3	4	4
r_{54}^{28}	$L(18,5)$	$\mathbb{Z}_{18}$	"	4	4
r_{56}^{28}	$S^3/(C_3 \times Q_{32}) = \mathbb{S}^3/(Q_{32} \times \mathbb{Z}_3)$	$\mathbb{Z}_2 \oplus \mathbb{Z}_6$	"	6	6
r_{65}^{28}	$L(17,5)$	$\mathbb{Z}_{17}$	"	4	4
r_{70}^{28}	$L(15,4)$	$\mathbb{Z}_{15}$	"	4	4
r_{71}^{28}	$L(14,3)$	$\mathbb{Z}_{14}$	"	4	4
r_{172}^{28}	$< 7,3,2 >= (\mathbb{S}^2, (2,1),(3,1),(7,-6))$	0	$SL_2\mathbb{R}$	7	7
r_{202}^{28}	$< 5,5,2 > 2 = (\mathbb{S}^2, (2,1),(4,1),(5,-4))$	$\mathbb{Z}_2$	$SL_2\mathbb{R}$	7	7
r_{203}^{28}	$< 7,3,2 > 5 = (\mathbb{S}^2, (2,1),(3,1),(7,-5))$	$\mathbb{Z}_5$	$SL_2\mathbb{R}$	7	7
r_{230}^{28}	$L(7,1)$	$\mathbb{Z}_7$	S^3	4	4
r_{280}^{28}	$TB\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$	$\oplus_2\mathbb{Z} \oplus \mathbb{Z}_2$	Nil	7	7
r_{402}^{28}	$S^3/(C_7 \times_i C_{12}) = \mathbb{S}^3/(Q_{28} \times \mathbb{Z}_3)$	$\mathbb{Z}_{12}$	S^3	6	6
r_{2314}^{28}	$S^3/(C_7 \times_i C_8) = \mathbb{S}^3/D_{56}$	$\mathbb{Z}_8$	"	6	6
r_{2418}^{28}	$S^3/(C_5 \times_i C_4) = \mathbb{S}^3/Q_{20}$	$\mathbb{Z}_4$	"	5	5