TABLE 2: EXPLANATORY NOTES

Table 2 summarizes details about all prime orientable 3-manifolds with *gem-complexity* 14, i.e. which admit crystallizations with 30 vertices and do not admit crystallizations with less than 30 vertices.

Each row of the table corresponds to a different manifold:

- The first column gives information about the *minimal gem* for the considered manifold M^3, that is the first element in the crystallization catalogue $\mathcal{C}^{30}$ representing M^3: more precisely, r_j^{30} denotes the j-th crystallization with 30 vertices belonging to the catalogue $\mathcal{C}^{30}$.
- The second column identifies M^3 via its JSJ decomposition or fibering structure.
- The third (resp. fourth) column contains the first homology group (resp. the geometric structure[1]) for M^3.
- The fifth column identifies M^3 within Matveev's tables of manifolds represented with spines of complexity ≤ 11 (see [M][2]); more precisely, c_x means that M^3 is the x-th element of Matveev's table of complexity c closed orientable 3-manifolds.

As far as the identification of M^3 (contained in the second column) is concerned, the following notations are used:

- $\mathbb{S}^3/G$ is the quotient space of $\mathbb{S}^3$ by the action of the group G; the involved groups are (direct products of) cyclic groups $\mathbb{Z}_n$ ($n \in \mathbb{Z}^+$), or groups of type $Q_{4n} =< x, y \,|\, x^2 = (xy)^2 = y^n >$, $D_{2^k(2n+1)} =< x, y \,|\, x^{2^k} = 1, y^{2n+1} = 1, xyx^{-1} = y^{-1} >$ ($n \in \mathbb{Z}^+$);
- $(F, (p_1, q_1), , \dots, (p_k, q_k), (1, b))$ is the Seifert fibered manifold with base surface F, twisting parameter b and k disjoint fibres, having (p_i, q_i), $i = 1, \dots, k$ as normalized parameters;
- for each matrix $A \in GL(2; \mathbb{Z})$ with $\det(A) = +1$, $TB(A) = T \times I/A$ is the orientable torus bundle over $\mathbb{S}^1$ with monodromy induced by A;
- for each matrix $A \in GL(2; \mathbb{Z})$ with $\det(A) = -1$, $(K \tilde{\times} I) \cup (K \tilde{\times} I)/A$ is the orientable 3-manifold obtained by pasting together, according to A, two copies of the orientable $I-$bundle over the Klein bottle K;

[1]Geometric structure, if any, is given according to [P.Scott, *The geometries of 3-manifolds*, Bull. London Math. Soc. **15** (1983), 401-487]; the symbol "-" is used to denote a non-geometric manifold.

[2]S.Matveev, *Table of closed orientable irreducible three-manifolds up to complexity 11*, available at WEB page: http://www.topology.kb.csu.ru/∼recognizer

- $H_1 \bigcup_A H_2$ is the graph manifold obtained by gluing Seifert manifold H_1 and Seifert manifold H_2 (whose base surfaces are either the annulus $\mathbb{A}$ or the disc $\mathbb{D}$) along their boundary tori by means of the attaching map associated to matrix A;

- $Q_i(p,q)$ denotes the manifold obtained as Dehn filling with parameters (p,q) of the hyperbolic manifold Q_i of finite volume and with a single cusp (see the table of manifolds with a single cusp in the SnapPea software by Weeks, available for anonimous FTP from www.geometrygames.org/SnapPea).

Remark:
Note that the two Sol manifolds of type $(K \tilde{\times} I) \cup (K \tilde{\times} I)/A$, with associated matrices $\begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix}$ and $\begin{pmatrix} -1 & -1 \\ 1 & 2 \end{pmatrix}$ respectively, have also the structure of (geometric) graph manifolds: they are $(\mathbb{D},(2,1),(2,1),(1,-1)) \bigcup_{\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}} (\mathbb{D},(2,1),(2,1),(1,-1))$ and

$(\mathbb{D},(2,1),(2,1),(1,0)) \bigcup_{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}} (\mathbb{D},(2,1),(2,1),(1,1))$ respectively.