

Details about analysis of all prime orientable 3-manifolds with *gem-complexity* 14 (i.e. which admit crystallizations with 30 vertices and do not admit crystallizations with less than 30 vertices) are shown in the following Table 2.

Each row of the table corresponds to a different manifold:

- The first column gives information about the *minimal gem* for the considered manifold M^3 , that is the first element in the crystallization catalogue \mathcal{C}^{30} representing M^3 : more precisely, r_j^{30} denotes the j -th crystallization with 30 vertices belonging to the catalogue \mathcal{C}^{30} .
- The second column identifies M^3 via its JSJ decomposition or fibering structure.
- The third (resp. fourth) column contains the first homology group (resp. the geometric structure¹) for M^3 .
- The fifth column identifies M^3 within Matveev's tables of manifolds represented by spines of complexity ≤ 11 (see [M]: S.Matveev, *Table of closed orientable irreducible three-manifolds up to complexity 11*, available at the Web page: <http://www.topology.kb.csu.ru/~recognizer>); more precisely, c_x means that M^3 is the x -th element of Matveev's table of complexity c closed orientable 3-manifolds.

As far as the identification of M^3 (contained in the second column) is concerned, the following notations are used:

- \mathbb{S}^3/G is the quotient space of \mathbb{S}^3 by the action of the group G ; the involved groups are (direct products of) cyclic groups \mathbb{Z}_n ($n \in \mathbb{Z}^+$), or groups of type $Q_{4n} = \langle x, y \mid x^2 = (xy)^2 = y^n \rangle$, $D_{2^k(2n+1)} = \langle x, y \mid x^{2^k} = 1, y^{2n+1} = 1, xyx^{-1} = y^{-1} \rangle$ ($n \in \mathbb{Z}^+$);
- $(F, (p_1, q_1), \dots, (p_k, q_k), (1, b))$ is the Seifert fibered manifold with base surface F , twisting parameter b and k disjoint fibres, having (p_i, q_i) , $i = 1, \dots, k$ as normalized parameters;
- for each matrix $A \in GL(2; \mathbb{Z})$ with $\det(A) = +1$, $TB(A) = T \times I/A$ is the orientable torus bundle over \mathbb{S}^1 with monodromy induced by A ;
- for each matrix $A \in GL(2; \mathbb{Z})$ with $\det(A) = -1$, $(K \tilde{\times} I) \cup (K \tilde{\times} I)/A$ is the orientable 3-manifold obtained by pasting together, according to A , two copies of the orientable I -bundle over the Klein bottle K ;
- $H_1 \bigcup_A H_2$ is the graph manifold obtained by gluing Seifert manifold H_1 and Seifert manifold H_2 (whose base surfaces are either the annulus \mathbb{A} or the disc \mathbb{D}) along their boundary tori by means of the attaching map associated to matrix A ;

¹Geometric structure, if any, is given according to [P.Scott, *The geometries of 3-manifolds*, Bull. London Math. Soc. **15** (1983), 401-487]; the symbol “-” is used to denote a non-geometric manifold.

- $Q_i(p, q)$ denotes the manifold obtained as Dehn filling with parameters (p, q) of the hyperbolic manifold Q_i of finite volume and with a single cusp (see the table of manifolds with a single cusp in the SnapPea software by Weeks, available for anonymous FTP from www.geometrygames.org/SnapPea).

minimal gem	prime orientable 3-manifold M^3	$H_1(M^3)$	geometry	position in [M]
r_{1203}^{30}	S^3/D_{80}	\mathbb{Z}_{16}	S^3	6 ₄₄
r_{1053}^{30}	S^3/D_{112}	\mathbb{Z}_{16}	S^3	6 ₄₈
r_{18250}^{30}	$S^3/(Q_{28} \times Z_5)$	\mathbb{Z}_{20}	S^3	6 ₄₉
r_{21444}^{30}	$S^3/(Q_{32} \times Z_5)$	$\mathbb{Z}_2 + \mathbb{Z}_{10}$	S^3	6 ₅₁
r_{1045}^{30}	$S^3/(P_{48} \times Z_{11})$	\mathbb{Z}_{22}	S^3	6 ₅₇
r_{1035}^{30}	$S^3/(P_{48} \times Z_5)$	\mathbb{Z}_{10}	S^3	6 ₅₆
r_{1040}^{30}	$S^3/(P_{48} \times Z_7)$	\mathbb{Z}_{14}	S^3	6 ₅₅
r_{19178}^{30}	$S^3/(P_{120} \times Z_{23})$	\mathbb{Z}_{23}	S^3	6 ₆₁
r_{17733}^{30}	$S^3/(P_{120} \times Z_{17})$	\mathbb{Z}_{17}	S^3	6 ₆₀
r_{1122}^{30}	$S^3/(P_{120} \times Z_{13})$	\mathbb{Z}_{13}	S^3	6 ₅₉
r_{21303}^{30}	$(S^2, (2, 1), (3, 1), (7, 3), (1, -1))$	\mathbb{Z}_{11}	$SL_2(\mathbb{R})$	7 ₁₂₀
r_{17842}^{30}	$(S^2, (2, 1), (3, 1), (8, 1), (1, -1))$	\mathbb{Z}_2	$SL_2(\mathbb{R})$	8 ₂₂₆
r_{21350}^{30}	$(S^2, (2, 1), (3, 1), (8, 3), (1, -1))$	\mathbb{Z}_{10}	$SL_2(\mathbb{R})$	7 ₁₂₇
r_{28623}^{30}	$(S^2, (2, 1), (3, 1), (9, 2), (1, -1))$	\mathbb{Z}_3	$SL_2(\mathbb{R})$	8 ₂₃₁
r_{44846}^{30}	$(S^2, (2, 1), (3, 1), (11, 2), (1, -1))$	0	$SL_2(\mathbb{R})$	8 ₂₄₃
r_{17755}^{30}	$(S^2, (2, 1), (4, 1), (5, 2), (1, -1))$	\mathbb{Z}_6	$SL_2(\mathbb{R})$	7 ₁₃₄

(Table 2 continues...)

minimal gem	prime orientable 3-manifold M^3	$H_1(M^3)$	geometry	position in [M]
r_{45301}^{30}	$(\mathbb{S}^2, (2, 1), (4, 1), (7, 2), (1, -1))$	\mathbb{Z}_2	$SL_2(\mathbb{R})$	8 ₂₇₁
r_{48748}^{30}	$(\mathbb{S}^2, (2, 1), (5, 1), (5, 1), (1, -1))$	\mathbb{Z}_5	$SL_2(\mathbb{R})$	8 ₂₈₃
r_{48763}^{30}	$(\mathbb{S}^2, (2, 1), (5, 2), (5, 2), (1, -1))$	\mathbb{Z}_{15}	$SL_2(\mathbb{R})$	7 ₁₃₈
r_{19485}^{30}	$(\mathbb{S}^2, (3, 2), (3, 2), (3, 2), (1, -1))$	$\mathbb{Z}_3 + \mathbb{Z}_9$	<i>Nil</i>	6 ₆₇
r_{15814}^{30}	$(\mathbb{S}^2, (3, 1), (3, 2), (3, 2), (1, -1))$	$\mathbb{Z}_3 + \mathbb{Z}_6$	<i>Nil</i>	6 ₆₆
r_{20091}^{30}	$TB \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$	$\mathbb{Z}_3 + \mathbb{Z} + \mathbb{Z}$	<i>Nil</i>	8 ₃₇₇
r_{56760}^{30}	$(T, (2, 1))$	$\mathbb{Z} + \mathbb{Z}$	$SL_2(\mathbb{R})$	9 ₉₀₂
r_{21193}^{30}	$(\mathbb{RP}^2, (2, 1), (3, 1))$	\mathbb{Z}_{24}	$SL_2(\mathbb{R})$	7 ₁₆₄
r_{21188}^{30}	$(\mathbb{RP}^2, (2, 1), (3, 2))$	\mathbb{Z}_{24}	$SL_2(\mathbb{R})$	7 ₁₆₅
r_{20090}^{30}	$TB \begin{pmatrix} -1 & 0 \\ 3 & -1 \end{pmatrix}$	$\mathbb{Z}_4 + \mathbb{Z}$	<i>Nil</i>	8 ₃₉₂
r_{56762}^{30}	$(K, (2, 1))$	$\mathbb{Z}_8 + \mathbb{Z}$	$SL_2(\mathbb{R})$	9 ₉₄₁
r_{17038}^{30}	$TB \begin{pmatrix} -4 & 1 \\ -1 & 0 \end{pmatrix}$	$\mathbb{Z}_6 + \mathbb{Z}$	<i>Sol</i>	8 ₃₉₃
r_{17043}^{30}	$TB \begin{pmatrix} 4 & -1 \\ 1 & 0 \end{pmatrix}$	$\mathbb{Z}_2 + \mathbb{Z}$	<i>Sol</i>	8 ₃₉₄
r_{56755}^{30}	$(\mathbb{A}, (2, 1), (1, -2)) \cup \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} (\mathbb{A}, (2, 1), (1, -2))$	$\mathbb{Z}_7 + \mathbb{Z}$	-	9 ₉₅₂
r_{56759}^{30}	$(\mathbb{A}, (2, 1), (1, -1)) \cup \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} (\mathbb{A}, (2, 1), (1, -1))$	$\mathbb{Z}_5 + \mathbb{Z}$	-	9 ₉₅₀

(Table 2 continues...)

minimal gem	prime orientable 3-manifold M^3	$H_1(M^3)$	geometry	position in [M]
r_{21476}^{30}	$(K \tilde{\times} I) \cup (K \tilde{\times} I) / \begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix}$	$\mathbb{Z}_2 + \mathbb{Z}_2 + \mathbb{Z}_4$	<i>Sol</i>	7 ₁₇₁
r_{45716}^{30}	$(K \tilde{\times} I) \cup (K \tilde{\times} I) / \begin{pmatrix} -1 & -1 \\ 1 & 2 \end{pmatrix}$	$\mathbb{Z}_4 + \mathbb{Z}_4$	<i>Sol</i>	7 ₁₆₉
r_{19144}^{30}	$(\mathbb{D}, (2, 1), (2, 1), (1, 0)) \cup \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} (\mathbb{D}, (2, 1), (3, 2), (1, 0))$	\mathbb{Z}_4	-	7 ₁₇₃
r_{18104}^{30}	$(\mathbb{D}, (2, 1), (2, 1), (1, 0)) \cup \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} (\mathbb{D}, (2, 1), (3, 2), (1, -1))$	\mathbb{Z}_{20}	-	7 ₁₇₅
r_{19251}^{30}	$(\mathbb{D}, (2, 1), (2, 1), (1, 0)) \cup \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} (\mathbb{D}, (2, 1), (3, 1), (1, -1))$	$\mathbb{Z}_2 + \mathbb{Z}_2$	-	8 ₄₁₀
r_{19087}^{30}	$(\mathbb{D}, (2, 1), (2, 1), (1, 0)) \cup \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} (\mathbb{D}, (2, 1), (3, 1), (1, -1))$	\mathbb{Z}_{28}	-	7 ₁₇₄
r_{11111}^{30}	$(\mathbb{D}, (2, 1), (2, 1), (1, 0)) \cup \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} (\mathbb{D}, (2, 1), (3, 1), (1, 0))$	\mathbb{Z}_4	-	7 ₁₇₂
r_{56897}^{30}	$Q_1(2, -3)$	$\mathbb{Z}_5 + \mathbb{Z}_5$	H^3	9 ₁₁₅₁
r_{45332}^{30}	$Q_4(2, -1)$	$\mathbb{Z}_3 + \mathbb{Z}_6$	H^3	9 ₁₁₅₃
r_{56912}^{30}	$Q_{10}(2, -1)$	$\mathbb{Z}_3 + \mathbb{Z}_9$	H^3	10 ₃₀₆₃

Table 2

The following statement summarizes the obtained classification of closed orientable prime 3-manifolds with gem-complexity 14:

Theorem I - *There exist exactly forty-one closed connected prime orientable 3-manifolds, which admit a coloured triangulation consisting of 30 tetrahedra, and do not admit a coloured triangulation consisting of less than 30 tetrahedra.*

Among them, there are:

- 10 elliptic 3-manifolds;
- 17 Seifert non-elliptic 3-manifolds (in particular, 2 torus bundles with Nil geometry);
- 2 torus bundles with Sol geometry;
- 2 manifolds of type $(K \tilde{\times} I) \cup (K \tilde{\times} I)/A$, i.e. the ones associated to matrices $\begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix}$ and $\begin{pmatrix} -1 & -1 \\ 1 & 2 \end{pmatrix}$, with Sol geometry; they are the geometric graph manifolds $(\mathbb{D}, (2, 1), (2, 1), (1, -1)) \cup \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} (\mathbb{D}, (2, 1), (2, 1), (1, -1))$ and $(\mathbb{D}, (2, 1), (2, 1), (1, 0)) \cup \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} (\mathbb{D}, (2, 1), (2, 1), (1, 1))$ respectively;
- 7 non-geometric graph manifolds;
- 3 hyperbolic Dehn-fillings (of the complement of link 6_1^3).