Details about analysis of the prime 3-manifolds represented in the catalogue $\mathcal{C}^{28}$ are shown in the following Table, where each row corresponds to a different manifold.

The first column of the Table contains Lins's notation for the first crystallization in $\mathcal{C}^{28}$ representing the considered 3-manifold: more precisely, r^i_j denotes the j-th crystallization with i vertices belonging to the catalogue.

The second column contains the name of the represented manifold according to Lins's notations[1], which is followed - when it is necessary - by the corresponding Matveev's description[2] and/or the associated Seifert structure.

For sake of completeness, we have also included both the first homology group and the geometric structure[3] of the manifold, which appear respectively in the third and fourth column.

minimal gem	**prime orientable 3-manifold $\mathbf{M^3}$**	$\mathbf{H_1(M^3)}$	**geometry**	**upper bound for $\mathbf{c'_{GM}(M^3)}$**	$\mathbf{c(M^3)}$
r^2_1	$\mathbb{S}^3$	0	S^3	0	0
r^8_1	$L(2,1)$	$\mathbb{Z}_2$	"	0	0
r^{12}_1	$L(3,1)$	$\mathbb{Z}_3$	"	0	0
r^{16}_1	$L(5,2)$	$\mathbb{Z}_5$	"	1	1
r^{16}_2	$L(4,1)$	$\mathbb{Z}_4$	"	1	1
r^{18}_1	$QUAT = \mathbb{S}^3/Q_8$	$\oplus_2\mathbb{Z}_2$	"	2	2
r^{20}_1	$L(8,3)$	$\mathbb{Z}_8$	"	2	2
r^{20}_2	$L(5,1)$	$\mathbb{Z}_5$	"	2	2
r^{20}_4	$S^3/ < 3,2,2 >= \mathbb{S}^3/Q_{12}$	$\mathbb{Z}_4$	"	3	3
r^{20}_5	$L(7,2)$	$\mathbb{Z}_7$	"	2	2
r^{20}_9	$\mathbb{S}^1 \times \mathbb{S}^2$	$\mathbb{Z}$	$S^2 \times \mathbb{R}$	0	0
r^{22}_1	$BINTET = \mathbb{S}^3/P_{24}$	$\mathbb{Z}_3$	S^3	4	4
r^{22}_2	$S^3/(C_3 \times_i C_8) = \mathbb{S}^3/D_{24}$	$\mathbb{Z}_8$	"	4	4

[1] Lins's notations (see [2]) are improved by the identification of five torus bundles arisen from [1].
[2] See Appendix 9.1 and Appendix 9.3 of [3].
[3] Geometric structure is given according to [5].

gem	3-manifold $\mathbf{M^3}$	$\mathbf{H_1(M^3)}$	geometry	$\mathbf{c_{GM}(M^3)}$	$\mathbf{c(M^3)}$
r^{24}_{1}	$EUCLID_0 = TB\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$	$\oplus_3\mathbb{Z}$	E^3	6	6
r^{24}_{2}	$BINDOD = \mathbb{S}^3/P_{120}$	0	S^3	5	5
r^{24}_{4}	$S^3/(C_5 \times_i C_8) = \mathbb{S}^3/D_{40}$	$\mathbb{Z}_8$	"	5	5
r^{24}_{5}	$EUCLID_1 = (K\tilde{\times}I)\cup(K\tilde{\times}I)/\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$	$\oplus_2\mathbb{Z}_4$	E^3	6	6
r^{24}_{6}	$EUCLID_3 = TB\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}$	$\oplus_2\mathbb{Z}_2 \oplus \mathbb{Z}$	E^3	6	6
r^{24}_{7}	$EUCLID_2 = TB\begin{pmatrix}0 & 1\\ -1 & -1\end{pmatrix}$	$\mathbb{Z} \oplus \mathbb{Z}_3$	E^3	6	6
r^{24}_{13}	$S^3/(C_3 \times Q_8) = \mathbb{S}^3/(Q_8 \times \mathbb{Z}_3)$	$\mathbb{Z}_2 \oplus \mathbb{Z}_6$	S^3	4	4
r^{24}_{14}	$L(9,2)$	$\mathbb{Z}_9$	"	3	3
r^{24}_{21}	$L(10,3)$	$\mathbb{Z}_{10}$	"	3	3
r^{24}_{22}	$L(11,3)$	$\mathbb{Z}_{11}$	"	3	3
r^{24}_{3}	$L(13,5)$	$\mathbb{Z}_{13}$	"	3	3
r^{24}_{28}	$BINOCT = \mathbb{S}^3/P_{48}$	$\mathbb{Z}_2$	"	5	5
r^{24}_{32}	$L(12,5)$	$\mathbb{Z}_{12}$	"	3	3
r^{24}_{33}	$L(6,1)$	$\mathbb{Z}_6$	"	3	3
r^{24}_{154}	$\mathbb{S}^3/Q_{16}$	$\oplus_2\mathbb{Z}_2$	"	4	4
r^{26}_{4}	$S^3/(Q_8 \times_3 C_{15}) = \mathbb{S}^3/(P_{24} \times \mathbb{Z}_5)$	$\mathbb{Z}_{15}$	"	6	5
r^{26}_{5}	$E_2(0,2) = (K\tilde{\times}I)\cup(K\tilde{\times}I)/\begin{pmatrix}1 & 1\\ 1 & 0\end{pmatrix}$	$\oplus_2\mathbb{Z}_4$	Nil	6	6
r^{26}_{6}	$S^3/(C_5 \times_i C_{12}) = \mathbb{S}^3/(Q_{20} \times \mathbb{Z}_3)$	$\mathbb{Z}_{12}$	S^3	5	5
r^{26}_{8}	$S^3/(Q_8 \times_3 C_9) = \mathbb{S}^3/P'_{72}$	$\mathbb{Z}_9$	"	5	5
r^{26}_{10}	$S^3/(C_7 \times < 5,3,2 >) = \mathbb{S}^3/(P_{120} \times \mathbb{Z}_7)$	$\mathbb{Z}_7$	"	6	6
r^{26}_{11}	$EUCLID_5 = TB\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$	$\mathbb{Z} \oplus \mathbb{Z}_2$	E^3	6	6
r^{26}_{13}	$E_2(2,1) = TB\begin{pmatrix}-1 & 0\\ -1 & -1\end{pmatrix}$	$\mathbb{Z} \oplus \mathbb{Z}_4$	Nil	6	6
r^{26}_{14}	$TB\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}$	$\oplus_2\mathbb{Z}$	Nil	6	6
r^{26}_{31}	$EUCLID_4 = TB\begin{pmatrix}1 & -1\\ 1 & 0\end{pmatrix}$	$\mathbb{Z}$	E^3	6	6
r^{26}_{65}	$S^3/(C_3 \times Q_{16}) = \mathbb{S}^3/(Q_{16} \times \mathbb{Z}_3)$	$\mathbb{Z}_2 \oplus \mathbb{Z}_6$	S^3	5	5

gem	3-manifold $\mathbf{M^3}$	$\mathbf{H_1(M^3)}$	geometry	$\mathbf{c_{GM}(M^3)}$	$\mathbf{c(M^3)}$
r_1^{28}	$[24, S_2^1] = (\mathbb{RP}^2, (2,1), (3,-1))$	$\mathbb{Z}_{24}$	$SL_2\mathbb{R}$	8	7
r_2^{28}	$[3, 5^2, S_6^5] = (\mathbb{S}^2, (3,1), (3,1), (5,-3))$	$\mathbb{Z}_3$	"	8	7
r_3^{28}	$[2^2, 3^2 \times 6, S_{10}^9] =$ $= (\mathbb{S}^2, (2,1), (2,1), (2,1), (3,-4))$	$\oplus_2\mathbb{Z}_2$	$SL_2\mathbb{R}$	8	7
r_5^{28}	$TB\begin{pmatrix} 0 & 1 \\ -1 & 3 \end{pmatrix}$	$\mathbb{Z}$	Sol	7	7
r_6^{28}	$[2^2 \times 4, S_1^4] = (K\tilde{\times}I) \cup (K\tilde{\times}I)/\begin{pmatrix} -1 & 0 \\ -1 & 1 \end{pmatrix}$	$\oplus_2\mathbb{Z}_2 \oplus \mathbb{Z}_4$	Nil	6	6
r_7^{28}	$S^3/(C_5 \times Q_8) = \mathbb{S}^3/(Q_8 \times \mathbb{Z}_5)$	$\mathbb{Z}_2 \oplus \mathbb{Z}_{10}$	S^3	5	5
r_9^{28}	$[3, 4^2, S_3^4] = (\mathbb{S}^2, (3,1), (3,1), (4,-3))$	$\mathbb{Z}_3$	$SL_2\mathbb{R}$	7	7
r_{10}^{28}	$TB\begin{pmatrix} 0 & 1 \\ -1 & -3 \end{pmatrix}$	$\mathbb{Z} \oplus \mathbb{Z}_5$	Sol	7	7
r_{13}^{28}	$L(21,8)$	$\mathbb{Z}_{21}$	S^3	4	4
r_{14}^{28}	$L(16,7)$	$\mathbb{Z}_{16}$	"	4	4
r_{19}^{28}	$[4^2, S_1^8] = (\mathbb{RP}^2, (2,1), (2,3))$	$\oplus_2\mathbb{Z}_4$	Nil	7	7
r_{27}^{28}	$S^3/(C_3 \times_i C_{16}) = \mathbb{S}^3/D_{48}$	$\mathbb{Z}_{16}$	S^3	5	5
r_{29}^{28}	$S^3/(C_3 \times_i C_{20}) = \mathbb{S}^3/(Q_{12} \times \mathbb{Z}_5)$	$\mathbb{Z}_{20}$	"	5	5
r_{33}^{28}	$L(13,3)$	$\mathbb{Z}_{13}$	"	4	4
r_{34}^{28}	$[3^2, S_1^3] = (\mathbb{S}^2, (3,2), (3,1), (3,-2))$	$\oplus_2\mathbb{Z}_3$	Nil	6	6
r_{41}^{28}	$L(19,7)$	$\mathbb{Z}_{19}$	S^3	4	4
r_{42}^{28}	$TB\begin{pmatrix} -1 & 0 \\ -2 & -1 \end{pmatrix}$	$\oplus_2\mathbb{Z}_2 \oplus \mathbb{Z}$	Nil	7	7
r_{49}^{28}	$L(11,5) = L(11,2)$	$\mathbb{Z}_{11}$	S^3	4	4
r_{54}^{28}	$L(18,5)$	$\mathbb{Z}_{18}$	"	4	4
r_{56}^{28}	$S^3/(C_3 \times Q_{32}) = \mathbb{S}^3/(Q_{32} \times \mathbb{Z}_3)$	$\mathbb{Z}_2 \oplus \mathbb{Z}_6$	"	6	6
r_{65}^{28}	$L(17,5)$	$\mathbb{Z}_{17}$	"	4	4
r_{70}^{28}	$L(15,4)$	$\mathbb{Z}_{15}$	"	4	4
r_{71}^{28}	$L(14,3)$	$\mathbb{Z}_{14}$	"	4	4
r_{172}^{28}	$< 7,3,2 >= (\mathbb{S}^2, (2,1), (3,1), (7,-6))$	0	$SL_2\mathbb{R}$	7	7
r_{202}^{28}	$< 5,5,2 > 2 = (\mathbb{S}^2, (2,1), (4,1), (5,-4))$	$\mathbb{Z}_2$	$SL_2\mathbb{R}$	7	7
r_{203}^{28}	$< 7,3,2 > 5 = (\mathbb{S}^2, (2,1), (3,1), (7,-5))$	$\mathbb{Z}_5$	$SL_2\mathbb{R}$	7	7
r_{230}^{28}	$L(7,1)$	$\mathbb{Z}_7$	S^3	4	4
r_{280}^{28}	$TB\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$	$\oplus_2\mathbb{Z} \oplus \mathbb{Z}_2$	Nil	7	7
r_{402}^{28}	$S^3/(C_7 \times_i C_{12}) = \mathbb{S}^3/(Q_{28} \times \mathbb{Z}_3)$	$\mathbb{Z}_{12}$	S^3	6	6
r_{2314}^{28}	$S^3/(C_7 \times_i C_8) = \mathbb{S}^3/D_{56}$	$\mathbb{Z}_8$	"	6	6
r_{2418}^{28}	$S^3/(C_5 \times_i C_4) = \mathbb{S}^3/Q_{20}$	$\mathbb{Z}_4$	"	5	5

Table 1

As far as Matveev's notation is concerned, the following conventions are used:

- $(F,(p_1,q_1),,\dots,(p_k,q_k))$ is the Seifert fibered manifold with base surface F and k disjoint fibres, having $(p_i,q_i),\ \ i=1,\dots,k$ as non-normalized parameters;

- for each matrix $A\in GL(2;\mathbb{Z})$ with $\det(A)=+1,\ \ TB(A)=T\times I/A$ is the orientable torus bundle over $\mathcal{S}^1$ with monodromy induced by A;

- for each matrix $A\in GL(2;\mathbb{Z})$ with $\det(A)=-1,\ \ (K\tilde{\times}I)\cup(K\tilde{\times}I)/A$ is the orientable 3-manifold obtained by pasting together, according to A, two copies of the orientable $I-$bundle over the Klein bottle K;

- $\mathbb{S}^3/G$ is the quotient space of $\mathbb{S}^3$ by the action of the group G; the involved groups are (direct products of) cyclic groups $\mathbb{Z}_n,\ n\in\mathbb{Z}^+$, or belong to the following list:

$$Q_{4n}=<x,y\,|\,x^2=(xy)^2=y^n>;$$
$$D_{2^k(2n+1)}=<x,y\,|\,x^{2^k}=1,y^{2n+1}=1,xyx^{-1}=y^{-1}>;$$
$$P_{24}=<x,y\,|\,x^2=(xy)^3=y^3,x^4=1>;$$
$$P_{48}=<x,y\,|\,x^2=(xy)^3=y^4,x^4=1>;$$
$$P_{120}=<x,y\,|\,x^2=(xy)^3=y^5,x^4=1>;$$
$$P'_{72}=<x,y,z\,|\,x^2=(xy)^2=y^2,zxz^{-1}=xy,z^9=1>.$$

As a consequence of our "translation" work on catalogue $\mathcal{C}^{28}$, Lins's classification of the encoded 3-manifolds is improved by a complete description of their topological structure:

Proposition *The sixty-nine closed connected prime orientable 3-manifolds which admit a coloured triangulation consisting of at most 28 tetrahedra are:*

- $\mathbb{S}^3$*;*
- $\mathbb{S}^2\times\mathbb{S}^1$*;*
- *the six Euclidean orientable 3-manifolds;*
- *twenty-three lens spaces;*
- *twenty-one quotients of* $\mathbb{S}^3$ *by the action of their finite (non-cyclic) fundamental groups;*
- *six (non euclidean) torus bundles* $TB(A)$:
  - *the Nil ones, with complexity 6, associated to matrices* $\begin{pmatrix}-1&0\\-1&-1\end{pmatrix}$ *and* $\begin{pmatrix}1&0\\1&1\end{pmatrix}$;[4]
  - *the Nil ones, with complexity 7, associated to matrices* $\begin{pmatrix}-1&0\\-2&-1\end{pmatrix}$ *and* $\begin{pmatrix}1&-2\\0&1\end{pmatrix}$;[5]
  - *the Sol ones, with complexity 7, associated to matrices* $\begin{pmatrix}0&1\\-1&3\end{pmatrix}$ *and* $\begin{pmatrix}0&1\\-1&-3\end{pmatrix}$;[6]

[4] In [3], they are denoted respectively by 6_{68} and 6_{69}.
[5] In [3], they are denoted respectively by 7^*_5 and 7^*_4, and in [4] by 162 and 163.
[6] In [3], they are are denoted respectively by 7^*_7 and 7^*_6, and in [4] by 167 and 168.

- *two Nil 3-manifolds of type* $(K\tilde{\times}I)\cup(K\tilde{\times}I)/A$*, with complexity 6, i.e. the ones associated to matrices* $\begin{pmatrix}1 & 1\\ 1 & 0\end{pmatrix}$ *and* $\begin{pmatrix}-1 & 0\\ -1 & 1\end{pmatrix}$*;* [7]

- *another Nil 3-manifold with complexity 6, i.e. the manifold with Seifert structure* $(\mathbb{S}^2,(3,2),(3,1),(3,-2))$*;* [8]

- *eight Seifert 3-manifolds with complexity 7:*

  - *the Nil 3-manifold* $(\mathbb{RP}^2,(2,1),(2,3))$; [9]
  - *the* $SL_2\mathbb{R}$ *3-manifold* $(\mathbb{RP}^2,\ (2,1),(3,-1))$,[10] *which may be also seen as* $D_2\cup(K\tilde{\times}I)/\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$ *(*D_2 *being the oriented Seifert manifold* $(D^2,(2,1),(3,2))$*, equipped with the coordinate system* (μ,λ)*,* μ *being a meridian of* $D^2\times\mathbb{S}^1$ *and* λ *being a fiber of the Seifert structure);*
  - *the* $SL_2\mathbb{R}$ *3-manifold* $(\mathbb{S}^2,\ (2,1),\ (2,1),\ (2,1),\ (3,-4))$,[11] *which may be also seen as* $D_2\cup(K\tilde{\times}I)/\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$;
  - *the* $SL_2\mathbb{R}$ *3-manifold* $(\mathbb{S}^2,(2,1),(3,1),(7,-6))$*;* [12]
  - *the* $SL_2\mathbb{R}$ *3-manifold* $(\mathbb{S}^2,(2,1),(4,1),(5,-4))$; [13]
  - *the* $SL_2\mathbb{R}$ *3-manifold* $(\mathbb{S}^2,(3,1),(3,1),(5,-3))$; [14]
  - *the* $SL_2\mathbb{R}$ *3-manifold* $(\mathbb{S}^2,(3,1),(3,1),(4,-3))$; [15]
  - *the* $SL_2\mathbb{R}$ *3-manifold* $(\mathbb{S}^2,(2,1),(3,1),(7,-5))$. [16]

References

[1] M.R.Casali, *Representing and recognizing torus bundles over* $\mathbb{S}^1$, Boletin de la Sociedad Matematica Mexicana (special issue in honor of Fico), **10 (3)** (2004), 89-106.

[2] S.Lins, *Gems, computers and attractors for 3-manifolds*, Knots and Everything **5**, World Scientific, 1995.

[7] In [3], they are denoted respectively by 6_{74} and 6_{72}.

[8] In [3] it is denoted by 6_{62}.

[9] In [3] it is denoted by 7^*_{11}, and in [4] by 158.

[10] In [3] it is denoted by 7^*_{13}, and in [4] by 159.

[11] In [3] it is denoted by 7^*_{18}, and in [4] by 155.

[12] In [3] it is denoted by 7^*_2, and in [4] by 166.

[13] In [3] it is denoted by 7^*_3, and in [4] by 164.

[14] In [4] it is denoted by 142.

[15] In [4] it is denoted by 140.

[16] In [4] it is denoted by 110.

[3] S.Matveev, *Algorithmic topology and classification of 3-manifolds*, Algorithms and Computation in Mathematics **9**, Springer, 2003.

[4] B.Martelli - C.Petronio, *Census 7*, Table of closed orientable irreducible 3-manifolds having complexity 7, available from the Web page http://www.dm.unipi.it/pages/petronio/public_html/files/3D/c9/c9_census.html

[5] P.Scott, *The geometries of 3-manifolds*, Bull. London Math. Soc. **15** (1983), 401-487.