ABOUT Γ-*class*

Γ-*class* is a program which implements the algorithm described in in [Casali M.R., Cristofori P., *A catalogue of orientable 3-manifolds triangulated by 30 coloured tetrahedra*, to appear], with respect to a fixed (finite) set $\bar{\mathcal{S}}$ of admissible sequences of elementary combinatorial moves: it yields, from any given list X of crystallizations, the automatic partition of the elements of X into equivalence classes, such that each class contains only crystallizations representing the same manifold.

In order to explain in details how the set $\bar{\mathcal{S}}$ used in the implementation is chosen, some notations have to be introduced.

First of all note that, given a rigid crystallization (Γ, γ), there is an obvious procedure which, starting from the code of (Γ, γ), yields a rigid crystallization $(\Gamma^<, \gamma^<)$ which is colour-isomorphic to (Γ, γ) and such that an ordering is induced in $V(\Gamma^<)$ by the so called *rooted numbering algorithm* generating the code (see [S.Lins, *Gems, computers and attractors for 3-manifolds*, Knots and Everything **5**, World Scientific, 1995]).

Let now (Γ, γ) be a rigid crystallization and suppose that in $V(\Gamma)$ the ordering of the vertices induced by the code of (Γ, γ) is fixed; given an integer $i \in \{1, 2, 3\}$, we denote by ϵ_i the sequence of cancellations of (m, n)-dipoles of type $\{0, i\}$ and dipoles, according to the following rules:

- $m, n < 9$ (this condition is necessary to bound the possible number of vertices of $\theta_i(\Gamma)$).[1]

- supposing $V(\Gamma) = \{v_1, \ldots, v_{2p}\}$, with vertex labelling coherent with the fixed ordering, the generalized dipoles of type $\{0, i\}$ are looked for and cancelled for increasing value of the integer $m \cdot n$ and by starting from vertex v_1 up to v_{2p}; this means that, if $\delta(v_i)$ (resp. $\delta'(v_j)$) is a $(m, n)-$ (resp. a $(m', n')-$) generalized dipole at vertex v_i (resp. v_j), then the cancellation of $\delta(v_i)$ is performed before the cancellation of $\delta(v_j)$ iff $m \cdot n < m' \cdot n'$, or ($m \cdot n = m' \cdot n'$ and $i < j$).

- after each generalized dipole cancellation, proper dipoles and ρ-pairs, if any, are cancelled in the resulting graph.

Moreover, we assume ϵ_0 to denote the void sequence of moves.

Let us define the set $S_3^0 = \{\mu = (\mu_0 = 0, \mu_1, \mu_2, \mu_3) \mid \mu \text{ is a permutation of } \Delta_3\}$. If S_3^0 is considered as a lexicographically ordered set, let $\delta^{(k)} = (\delta_0^{(k)} = 0, \delta_1^{(k)}, \delta_2^{(k)}, \delta_3^{(k)})$ ($k \in \{1, 2, \ldots, 6\}$) denote the k-th element of S_3^0 .

For each $k \in \{1, 2, \ldots, 6\}$ and for each $i \in \Delta_3$, we set

$$\ll \delta_i^{(k)} \gg = \epsilon_{\delta_i^{(k)}} \circ \epsilon_{\delta_{i-1}^{(k)}} \circ \ldots \circ \epsilon_{\delta_0^{(k)}}.$$

[1] Cancellation of a generalized dipole increases the number of vertices, but dipoles are frequently created as a consequence, and their further cancellation allows to decrease the number of vertices.

Let us now set

$$\begin{aligned}\bar{\mathcal{S}} =& \{\ll \delta_i^{(k)} \gg \ / \ k \in \{1,2,\dots,6\} \text{ and } i \in \Delta_3\} \cup \\ & \cup \{\ll \delta_i^{(k)} \gg \circ \ll \delta_3^{(k-1)} \gg \circ \cdots \circ \ll \delta_3^{(1)} \gg \ / \ k \in \{2,\dots,6\} \text{ and } i \in \Delta_3\}\end{aligned}$$

The Γ-*class* program accepts as input a text file of codes of rigid crystallizations having the same bipartition type and asks to specify whether they are bipartite or not. Each row of the input file is in the form

Number_of_vertices string_of_code

At the end of the row a name for the represented manifold can be added, writing a " ∗ " first.

Example:
16 CABFDEHGHEDCBGFAGDFBAHEC
18 DABCGEFIHIHFEDCBGAHEGIBACFD∗L(5,2)

The allowed maximum number of vertices is 150.

Program Γ-*class* produces two text files:

- **classes.txt**: contains the list of the equivalence classes with respect to $\bar{\mathcal{S}}$ (each crystallization being represented by the number of its row in the input file), with the possible addition of names for the represented manifolds. Furthermore, if the function h takes value different from zero for a crystallization $\bar{\Gamma}$, the value $h(\bar{\Gamma})$ is displayed after the number corresponding to $\bar{\Gamma}$.

  Example:

  Class n. 1

  1 , 2 ($h = 1$),

  Manifold: ∗S3

  states that the first crystallization represents $\mathbb{S}^3$, while the second represents $\mathbb{S}^1 \times \mathbb{S}^2$.

- **archive.txt**: contains the list of codes as in the input file, each followed by the name of the represented manifold, if it has been identified (by means of known catalogues or connected sum splitting), or by the string " $*[n]$ ", where n is the number of the (unknown) class which the crystallization belongs to.

The connected sums checking part of program Γ-*class* needs two text files O.ARC and NO.ARC containing lists of known codes (each followed by the name of the represented manifold), which the program can use to recognize the summands arising from the splitting of a given crystallization.

The two files attached, as basic tool, to the program contain the catalogues $\mathbf{C}^{(28)}$ and $\mathbf{C}^{\tilde{(}26)}$ listing respectively the codes of all bipartite rigid crystallizations up to 28 vertices and non-bipartite ones up to 26 vertices.

Obviously the user can add to O.ARC (resp. NO.ARC) any rigid bipartite (resp. non-bipartite) crystallization by means of its code followed by a name (of length less than 50 characters) helping the identification.